Quiz 1: Graded on Crowdmark for most people
(others - email today)

Last time: Derivative →

Power Rule $x^n \longrightarrow n x^{n-1}$

Ex. $x^2 \longrightarrow 2x$

$\sqrt{x} = x^{1/2} \longrightarrow \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$ax^2+bx+c \longrightarrow 2ax + b$

Product Rule $f(x)g(x) \longrightarrow f'(x)g(x) + f(x)g'(x)$

Ex. $x^2 \sin x \longrightarrow 2x \sin x + x^2 \cos x$ $\quad((\sin x)' = \cos x)$

(Quotient Rule)

Today:

Chain Rule $f(g(x)) \longrightarrow f'(g(x)) \cdot g'(x)$

Ex. $f(x) = x^2+5 \longrightarrow 2x$

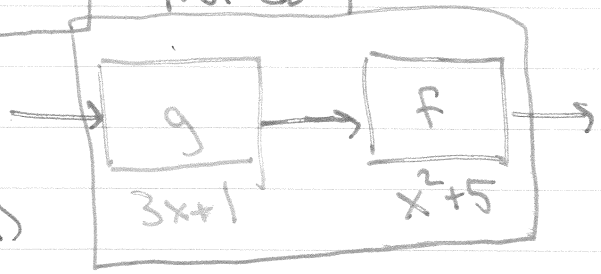
$g(x) = 3x+1 \longrightarrow 3$

$(3x+1)^2+5 \longrightarrow \boxed{2(3x+1)\cdot 3}$

+ Linear Approximation

Composition of Functions.

$f(g(x))$ means "first do g" "then do f" $f \circ g$



Q: $f(x) = x^2+5$, $g(x) = 3x+1$,

$f(g(x)) = ?$

$f(g(x)) =$ **A)** $(3x+1)^2+5$ C) $(x^2+5)(3x+1)$

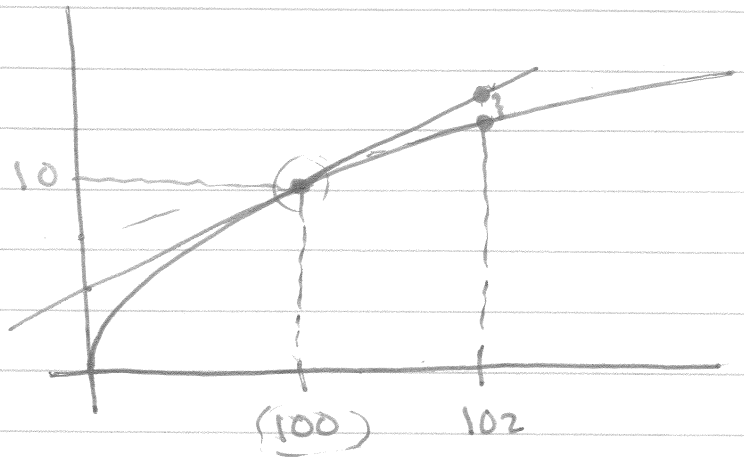
$g(f(x))$ **B)** $3(x^2+5)+1$ D) x^2+3x+6

E) Don't know, explain

$f \circ g \neq g \circ f$

Linear Approximation

Q: How to approximate $\sqrt{102}$? $\sqrt{100} = 10$



$f(x) = \sqrt{x} = x^{1/2}$

- At $x=100$ we know $\sqrt{x}$ AND can calculate $f'(x)$.
- But we can't do this at any other values of x

Linear Approx: Write down the eqn of the tangent line at $x=100$ $y = f_1(x)$

- Plug in $f_1(102)$.

→ How?
- It passes thru $(100, 10)$
- Its slope is $f_1'(100)$. $f'(x) = \frac{1}{2\sqrt{x}}$

$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

Point-Slope:

$$y - 10 = \frac{1}{20}(x-100) = \frac{1}{20}x - 5$$

In general. Pt (x_0, y_0), Slope m $\rightsquigarrow$ $y - y_0 = m(x - x_0)$

- Plug $x=102$. $y - 10 = \frac{1}{20}(\underbrace{102}_{\text{point we want}} - 100) = \frac{1}{20}(2) = \frac{1}{10}$

$$y = 10.1$$

Linear approximation to $\sqrt{102}$

Overestimate because $f(x) = \sqrt{x}$ $f''(x) < 0$